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Teaching geometry at the Barcelona Royal Academy of Mathematics (1720-1803)
Ensino de geometria na Academia Real de Matemática de Barcelona (1720-1803)

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Abstract: The article constitutes a study on the Teaching of Geometry at the Royal Academy of Mathematics in Barcelona between 1720 and 1803. The contribution seeks to reflect on the research of technique and art in the universe of treatises on Geometry and its circulation with respect to knowledge applied in the construction of buildings. In this sense, the research reflects how mathematical knowledge was fundamental for the architecture of fortifications and, above all, for a culture of knowledge that reflected in the architectural spatial vision of the Hispanic world.

Keywords: Fortification Architecture; Geometry; Royal Academy of Mathematics; B

Resumo: O artigo constitui um estudo sobre o Ensino de Geometria na Academia Real de Matemática de Barcelona entre 1720 a 1803. A contribuição busca refletir sobre a pesquisa da técnica e da arte no universo dos tratados de Geometria e a sua circulação no que tange aos conhecimentos aplicados na construção de edificações. Neste sentido a pesquisa reflete como o conhecimento matemático era fundamental para a arquitetura de fortificações e, sobretudo para uma cultura do conhecimento que refletia na visão espacial arquitetônica do mundo hispânico.

Palavra-chaves: Arquitetura de Fortificações; Geometria; Academia Real de Matemática; Barcelona;

Introduction

The Barcelona Royal Academy of Mathematics was founded on January 22nd, 1700 by virtue of a Royal Decree of the King Felipe V. The Academy operated until October 1705 and then closed for 15 years. It was reopened in 1720 under the direction of Mateo Calabro (1720-1738), an Italian military engineer who was first replaced by Pedro de Lucuze (1738-1779), then by Miguel Sánchez Taramas (1779-1789), and finally by Félix de Arriete (1789-1793). The decline of the Academy became evident during the end of the 18th century, until it reached a crisis that led it to its closure in 1803. However, it was during the second of its opening periods, while directed by Pedro de Lucuze, when the Academy consolidated its role as the sole teaching center for military engineers throughout Spain, coinciding with an intense activity in the construction of fortifications and

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the theoretical production of its professors. Overall, the academy was undoubtedly an institution permeable to cultural determinations from other European institutions. At the Academy, scientific and technical knowledge was elaborated and transmitted in accordance with more or less formal spatial and constructive models.

Purely in the academic sphere, the so-called "mathematical course" was rigorously taught during the years when Lucuze directed the institution. This course comprised of eight treatises that must be recorded by the students into manuscript notebooks. These treatises were always assigned in the same order, which can be verified by consulting copies in libraries in Madrid and Barcelona. Through these copies, it is possible to see the influence of authors such as Ozanam (1700), Fernández de Medrano (1708), Tosca (1712), and especially Belidor (1725), who were clear pioneers of the themes related to arithmetic, geometry, static, and civil engineering.

The *Mathematical course* created by Lucuze was constituted by the eight treatises in compliance with a royal ordinance dated 1739: 1. Arithmetic, 2. Elemental Geometry, 3. Practical Geometry, 4. Fortification, 5. Artillery, 6. Cosmography, 7. Static (with an appendix dedicated to optics), and 8. Civil architecture. As an overview, this mathematical course was an initiation for engineers rather than an actual tool in the conception of architectural works. Through the course, a series of problems were formulated with progressive difficulty to provide the engineers with a simplified mathematical knowledge, but enough to allow them to conduct discourse and projects.

Treatises devoted to geometry

In the first treatise of the mathematical course by Lucuze, which was dedicated to arithmetic, the students at the Barcelona Royal Academy of Mathematics learned whole numbers and fractions, basic operations (addition, subtraction, multiplication, and partitioning), rates and proportions, and powers and roots, as well as a brief theory of progressions. Having learned these items, the students would then progress to Elemental Geometry, referred to as "speculative geometry," which was essentially Euclidian Elements. This was taught through a treatise divided into six chapters and an appendix. The treatise was preceded by an introduction and a set of basic definitions.

Book I dealt with straight lines and types of angles; Book II dealt with the properties of triangles and parallelograms; Book III explained the properties of straight lines in relation to circles; Book IV examined ratios and proportions for flat shapes; Book V dealt with solids on the whole; and Book VI explained platonic solids, such as pyramids, cylinders, and spheres. Finally, an appendix was dedicated to conic sections, such as parabolas, ellipses, and hyperbolas.

This brief treatise dedicated to speculative geometry provides evidence of the relationship between the mathematical course by Lucuze and a book written by B.F. Belidor, entitled *Nouveau Cours de Mathematique* (1725). The first part of this course also dealt with geometry. The course was divided into eight books and included an annex dedicated to conic sections.

Immediately upon completing the study of speculative geometry, students went on to Treatise III, which was devoted to trigonometry and "practical geometry." This treatise was divided into eight books, following the same order as the *Nouveau Cours* by Belidor. Book I explained the use of the "trigonometric canon" and logarithms with the resolution of triangles and rectangles; Book II explored the construction of flat shapes; Book III focused on the registration and constituency of straight line figures in circles; Book IV dealt with their proportion, increased, decrease, and transformation; Book V explained the use of the most common instruments for longimetry and altimetry; Book VI dealt with surfaces dimension; Book VII examined the calculation of volumes; and Book VIII dealt with levelling. In all tables of contents, the influence of Belidor's and Ozanam (1700) was evident.

The speculative and practical geometry courses represented 31.3% of the mathematical course devoted to each of the eight treatises.

The pedagogical resources employed by the professors and contained in the manuscript texts were simple, and clear and precise definitions about each of the terms were always at the beginning of each class. Thereafter, and along the lines of Euclid, the theorems were stated prior to their demonstration. To conclude, the class in-

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cluded the corollaries and scholia, which students carefully copied into their notebooks. Finally, the professor suggested a series of practical problems that were developed in class.

The images accompanying these two geometric treatises provide clear examples of the practical applications for engineers in training. All students copied these images from their professors with fidelity.

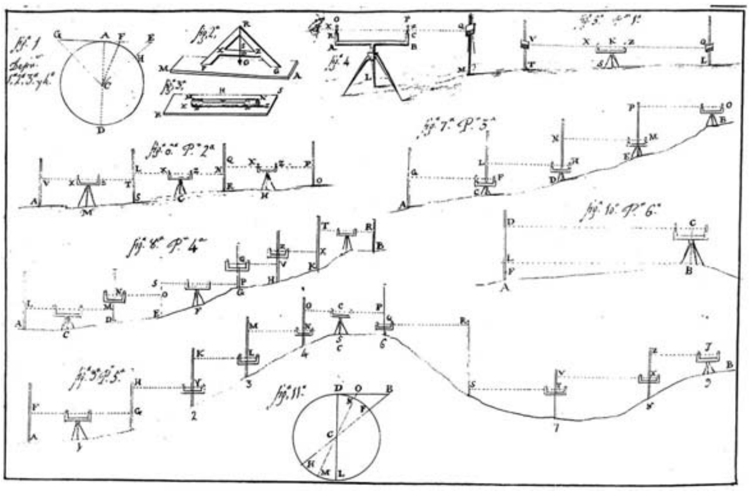

FIGURE 1. Sheet 1 from the 3rd *Treatise on Practical Geometry*.
Mss (not numbered) from the Library at the Montjuic Military Museum, Barcelona.

Figure 1 represents a simple exercise of an altimetry survey of a terrain with simple measuring instruments, reproducing examples previously developed by Belidor in his 1725 book. Figure 2 contains several exercises on determining horizontal and vertical distances from the theorems of triangle proportions. These exercises recurrently appeared in various printed military architecture treatises.

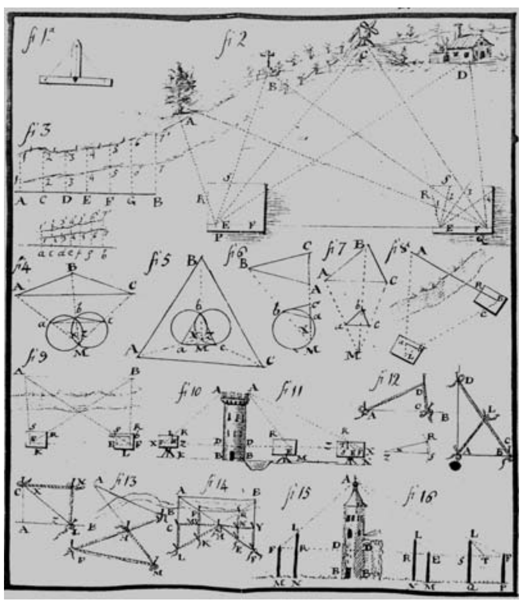

FIGURE 2. Sheet 2 from the 3rd *Treatise on Practical Geometry*.
Mss (not numbered) de la Library at the Montjuic Military Museum, Barcelona.

Once the initial courses were completed, students were introduced to the theory of regular and irregular fortification, offensive and campaign fortification contained in the 4th treatise, where Lucuze was detached from Belidor's authority, because Belidor did not autonomously develop the theme of fortification in any of his books. There are, however, clear references to the work of Vicente Tosca, author of *Compendio Matemático*

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(1712), who clearly supported the work of M. Dechales, *L'art de fortifier* (1677).

Who was responsible for the order of the courses? For centuries, the most empirical treatises in architecture and engineering indicated the importance of mathematics and its role in knowledge. From this point of view, it was common to first read about construction, an arithmetic and geometric introduction to which a brief trigonometric description was added on occasion.

The need for this mathematical support can be explained, in part, by the desire to participate in the prestige of this science to enhance the social estimation of the constructor. Furthermore, as part of the academic world, this link may be understood as a methodological assimilation that adopted the systematic rigor and the theoretical content of arithmetic and geometry, while the doctrine of the construction of arches and vaults was explained within an exposition style that implied demonstrating theorems and corollaries.

In the specific field of military architecture, that relationship was accentuated because the work to be constructed was conceived as a concrete and material verification of an abstract geometric figure. In effect, a building emerged from a geometric figure as a space limited by a structure of lines and surfaces to finally constitute a determined volume. Hence, the drawing of the fortification became a result of scientific reflection more than artistic imagination.

Geometry for the fortification

It is clear that in the mathematical course taught at the Barcelona Royal Academy, geometry was taught through two initial courses subsequent to arithmetic. These courses preserved the already classical division between speculative geometry and practical geometry. The first course was anchored in Euclidean tradition, and the latter sought to encompass operational aspects.

The two geometry courses aimed to construct a knowledge base for students to quickly be able to address the drawing of fortifications through a series of complex exercises related to the geometric construction of its traces, whether they were regular or irregular, explained in detail in the following course dedicated to fortification. This 4^{th} treatise was a practical text and was an exercise on the geometric problems raised by a type of construction that, like military construction, was oversized by a desire for functionality in purpose and performance.

Geometry already determined all polemology, making fortification an experimental and practical science that was kept in the hands of some scholars who received and imparted a regulated science that constituted a domain of knowledge.

Geometry also had to be taken to practice on the terrain and was aided by the instruments that provided its symmetry, planimetry, and stereotomy. However, iconography was the most used representation system for tracing the floors. This was also where the geometry lessons assimilated during the initial courses were put into practice. By carrying out these traces, engineers seem to inherit what it used to be the core of the work for architects, as was pointed out by the Spanish Covarrubias (1611) some years before: "provide the traces of the buildings and make the floor plans, training him first in his understanding (f. 84)."

To clearly understand how the fortified trace exercise put the geometric knowledge of the engineer in training into practice, it is worth reconstructing four of the problems that appeared in one of the treatises as part of the class notes from the mathematical course by Lucuze (6th *Treatise VI, of the Fortification*, Mss not numbered, Library at the Montjuic Military Museum, Barcelona). The purpose of this was to show the increasing degree of complexity with which the exercises were explained, making use only of the compass and set-square and all applied to concrete cases related to floor plan construction of fortifications.

Problem 1: Divide a straight line (AB) into equal parts.

This exercise was considered as introductory in the treatise corresponding to fortification. The purpose of this exercise was that the engineer could establish proportional relationships between different drawings needed to be constructed. Prior to their formulation, there was a mention of the use of a graphic scale (or *petipié*) that had to accompany the floors and sections of the fortifications. The operation could be fulfilled in five steps, all through the use of a compass and a ruler.

Operation: Trace an auxiliary line a bit longer than line AB and, by using the compass, divide it into five parts with M and N being its extremes. On the MN line, describe an equilateral triangle, MON. From point O, draw the hidden lines OF and OE which should cut the extremes of the five segments. Thus, we have an instrument that permits dividing a line easily and exactly. Next, take the compass to the given AB line and with the compass opening, place one end of the compass on point O and with the other end indicate points X and Z on the sides OM and ON, extremes of the straight line that would be divided into five parts. If the line to be divided were bigger than the MN line of the instrument, then it would be necessary to extend the lines OM and ON.

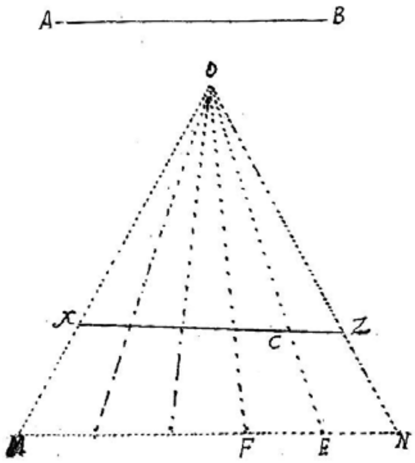

FIGURE 3. Sheet from the *6th Treatise on Fortification* in which Problem 1 was developed. Mss (not numbered) from the Library at the Montjuic Military Museum, Barcelona.

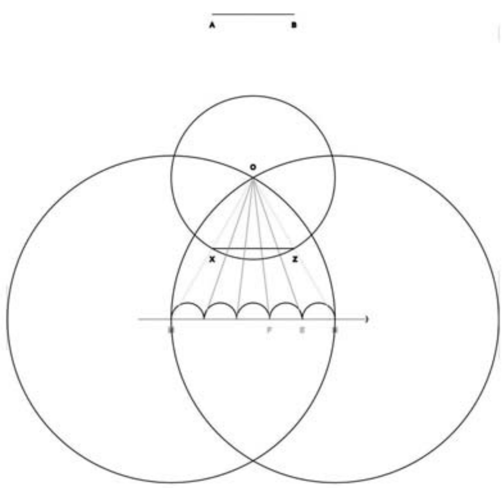

FIGURE 4. Traces of all the graphic operations necessary to develop Problem 1 (author's rendering).

Problem 2: Trace the master or main line for any fortified polygonal whose side is 180 toises.

Herein, Bauban's "Marshal Method" was used. This method seeks to trace the master line of a fortified square polygonal line whose side was previously determined to be 180 Castilian toises. By developing this exercise, the engineer in training was able to make similar traces for fortified constructions of up to 12 sides.

Operation: Line AB is one of the sides to be fortified. Divide it into two equal parts at the point C and, from it, raise the perpendicular CD with a length of 22.5 toises, because AB is the side of a square. If the figure was not a square, then the perpendicular CD would be 25 toises (pentagon) or 30 toises (six sides or more). From points A and B, indeterminate lines are drawn passing through point D and on them are marked the BE and AF fronts, each 50 toises long. Make the center at point E and with the compass open at EF, indicate point P on line BD. With the same compass opening, making the center at point F, indicate point Q. On line AD, draw the PQ curtain and the QE and PF flanks, along with the BE and AF fronts. The same operation should be done on each of the remaining three sides.

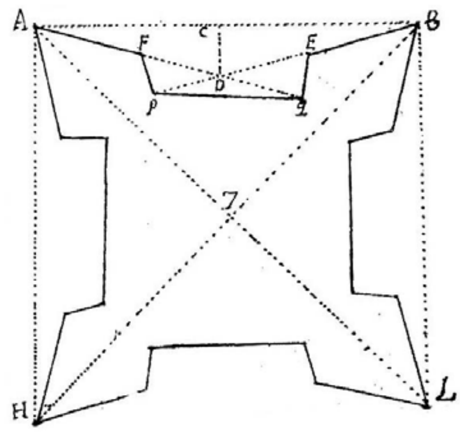

FIGURE 5. Sheet from the 6th *Treatise on Fortification* in which Problem 2 was developed.
Mss (not numbered) from the Library at the Montjuic Military Museum, Barcelona.

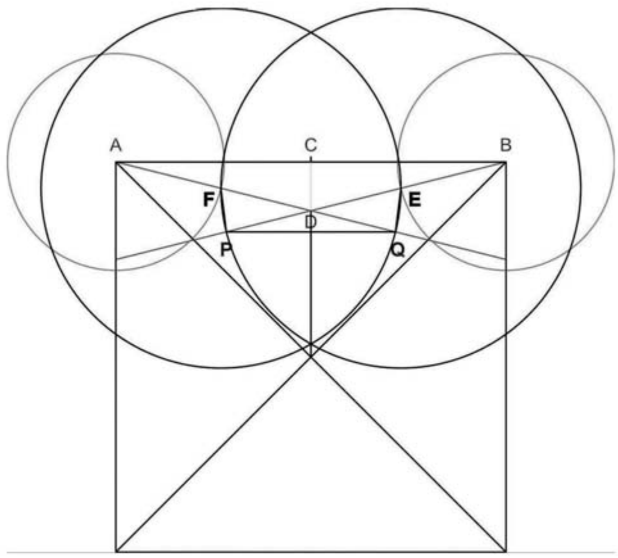

FIGURE 6. Traces of all the graphic operations necessary to develop Problem 2 (author's rendering).

Problem 3: Fortify the ABCD square whose inner side is 115 toises.

Contrary to the previous problem, this problem required a line to be traced starting from the dimension of the inner side. For the 18th century, this was of 115 Castilian toises.

Operation: On the AB side, form the two demigorges AE and FB that, according to fortification tables, must be 19 toises. From points E and F, take the flank angles, each of 95 degrees. Take each of the 20-toise flanks, also according to the tables and trace the defences to vortices V and S.

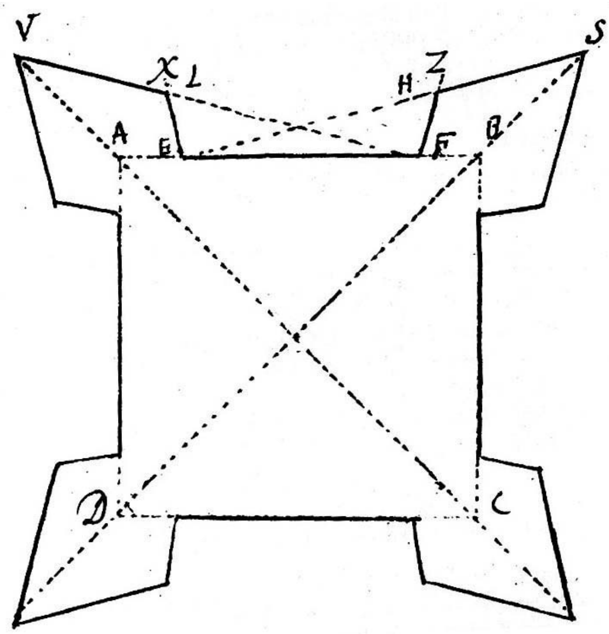

FIGURE 7. Sheet from the 6*th* *Treatise on Fortification* in which Problem 3 was developed.
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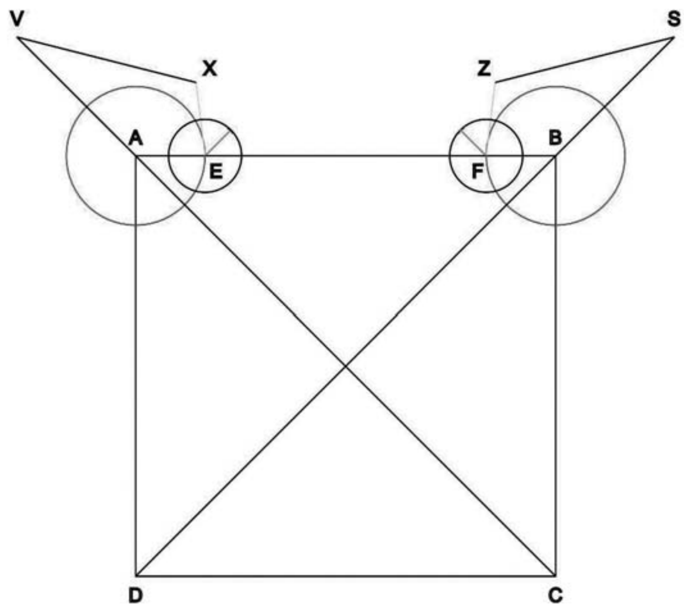

Barcelona.

FIGURE 8. Traces of all the graphic operations necessary to develop Problem 3 (author's rendering).

Problem: Construct the orillons and the curved flank of a bulwark.

Because Vauban had suppressed straight flanks, Spanish engineers found a way to make them curved from a more complex geometric game than what was developed in previous exercises. In this problem, the orillons protect the curved flanks where canons were placed. In this way, they were protected from direct enemy attack.

Operation: Divide the EF straight flank into three equal parts on the X and Z points. Take the 10-toise to 12-toise BT distance from point B on the BR front. Take the 5-toise FS distance from point T through points X, V and from point F over the BE line of defence. Take the same distance and from the X point go to O over the XV line. Make an equilateral triangle over SO or indicate the N vortex. To form the orillon, divide EX into two equal parts on point L. At point L, raise a long perpendicular line at discretion. On point E and in front

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of AE raise another perpendicular that intersects the other one at point G. Make a center on point G and, with the compass open at interval GE, describe the EKX arch and the orillon will be described.

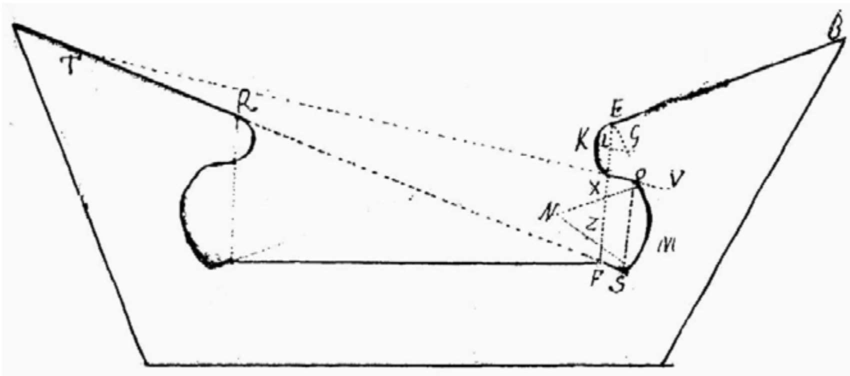

FIGURE 9. Sheet from the 6th *Treatise on Fortification* in which Problem 4 was developed.
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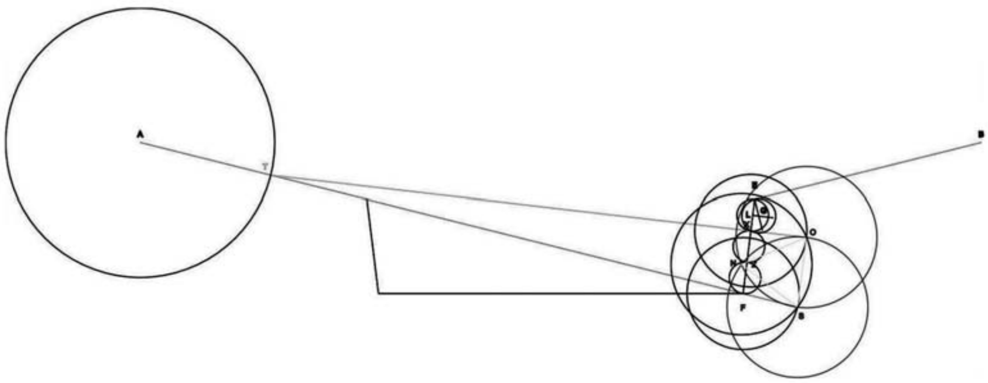

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FIGURE 10. Traces of all the graphic operations necessary to develop Problem 4 (author's rendering).

Conclusions

In the Barcelona Royal Academy of Mathematics, through the contents of the mathematical course taught, the most diverse fortification treatises printed in Europe during the 16th, 17th and 18th centuries clearly converged. There was the steadfast intention of training a corps of Spanish military engineers in tune with the most recent schools of technical and philosophical thought.

The teaching of geometry at this institution appears stripped of the symbolic character that dominated it for years in the form of guidelines that were simple but, above all, transmissible. However, its application to the utilitarian demands of military engineers and architects, especially the design and construction of fortified establishments, was important and required practical exercises capable of overcoming any degree of abstraction. The use of geometry was therefore confined to the representation of traces with eminently pragmatic purposes by the first Spanish engineers.

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